M1 S14(R)

1.

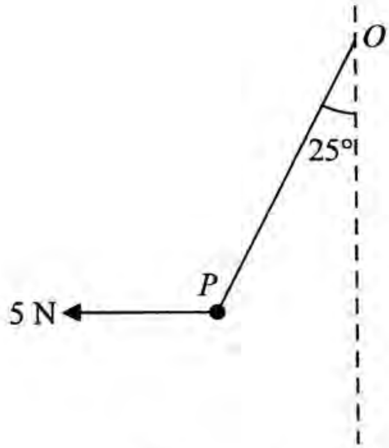


Figure 1

A particle P of weight W newtons is attached to one end of a light inextensible string. The other end of the string is attached to a fixed point O. A horizontal force of magnitude 5 N is applied to P. The particle P is in equilibrium with the string taut and with OP making an angle of 25° to the downward vertical, as shown in Figure 1.

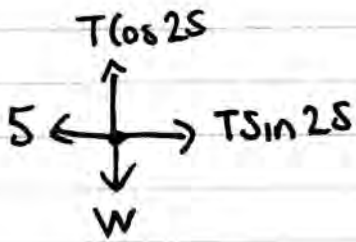
Find

(a) the tension in the string,

(3)

(b) the value of W.

(3)



$\overrightarrow{R} = 0 \quad 5 = T\sin 25 \quad \therefore T = \underline{11.8\text{N}}$

$R(\uparrow) = 0 \quad W = T\cos 25 \quad \therefore W = \underline{10.7\text{N}}$

2. Two forces $(4\mathbf{i} - 2\mathbf{j})$ N and $(2\mathbf{i} + q\mathbf{j})$ N act on a particle P of mass 1.5 [illegible] res[illegible] these two forces is parallel to the vector $(2\mathbf{i} + \mathbf{j})$.

(a) Find the value of q.

[illegible])

At time $t = 0$, P is moving with velocity $(-2\mathbf{i} + 4\mathbf{j})$ m s^{-1}.

(b) Find the speed of P at time $t = 2$ seconds.

(6)

a) $Rf = \begin{pmatrix} 4 \\ -2 \end{pmatrix} + \begin{pmatrix} 2 \\ q \end{pmatrix} = \begin{pmatrix} 6 \\ q-2 \end{pmatrix} = \lambda \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ $\quad 6 = 2\lambda \therefore \lambda = 3$

$\Rightarrow q - 2 = 3$

$\underline{\underline{q = 5}}$

b) $Rf = ma$ $\quad \begin{pmatrix} 6 \\ 3 \end{pmatrix} = 1.5a \quad \therefore a = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$

$v = u + at \quad v = \begin{pmatrix} -2 \\ 4 \end{pmatrix} + \begin{pmatrix} 4 \\ 2 \end{pmatrix} \times 2 = \begin{pmatrix} 6 \\ 8 \end{pmatrix}$

$\text{speed} = \sqrt{6^2 + 8^2} = \underline{\underline{10 \text{ ms}^{-1}}}$

3. A car starts from rest and moves with constant acceleration along a straight horizontal road. The car reaches a speed of V m s^{-1} in 20 seconds. It moves at constant speed V m s^{-1} for the next 30 seconds, then moves with constant deceleration $\frac{1}{2}$ m s^{-2} until it has speed 8 m s^{-1}. It moves at speed 8 m s^{-1} for the next 15 seconds and then moves with constant deceleration $\frac{1}{3}$ m s^{-2} until it comes to rest.

(a) Sketch, in the space below, a speed-time graph for this journey. **(3)**

In the first 20 seconds of this journey the car travels 140 m.

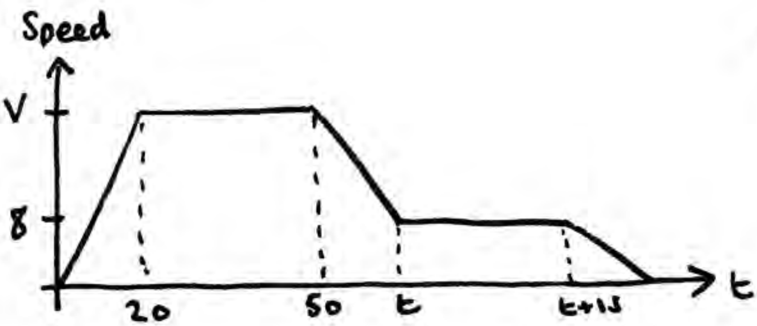
Find

(b) the value of V, **(2)**

(c) the total time for this journey, **(4)**

(d) the total distance travelled by the car. **(4)**

a)



b) 140 V 20 $\therefore 10V = 140$

$V = 14$

c) $\boxed{420}$ 14 30   6 x $\frac{6}{x} = \frac{1}{2}$ $\therefore x = 12$ 6 12 $\frac{6 \times 12}{2} = 36$ $\boxed{96}$ 8 12

8 $\boxed{120}$ 15   8 y $\frac{8}{y} = \frac{1}{3}$ $\therefore y = 24$ 8 24 $\frac{8 \times 24}{2} = 96$

total ~~time~~ distance $= 140 + 420 + 36 + 96 + 120 + 96 = 908$m

total time $= 50 + 12 + 15 + 24 = 101$sec

4. At time $t = 0$, a particle is projected vertically upwards with speed u fro[...]nt A. [...] particle moves freely under gravity. At time T the particle is at its maxi[...] above A.

(a) Find T in terms of u and g.

(b) Show that $H = \dfrac{u^2}{2g}$ **(2)**

The point A is at a height $3H$ above the ground.

(c) Find, in terms of T, the total time from the instant of projection to the instant when the particle hits the ground. **(4)**

$s = H$
$u = u$
$v = 0$
$a = -9.8$
$t = T$

a) $v = u + at$

$0 = u - 9.8T \quad \therefore \quad T = \dfrac{u}{g}$

b) $s = \dfrac{(u+v)t}{2} = \dfrac{(u+0)T}{2} = \frac{1}{2}T \times u$

$\therefore H = \frac{1}{2}\frac{u}{g}u = \dfrac{u^2}{2g}$

c) $s = -3H$
$u = Tg$
v
$a = -g$
t

$s = ut + \frac{1}{2}at^2$

$-3H = Tgt - \frac{1}{2}gt^2$

$-3H = \dfrac{-3u^2}{2g} = \dfrac{-3T^2g^{\cancel{2}}}{2\cancel{g}} = \dfrac{-3T^2g}{2}$

$\therefore -\frac{3}{2}T^2\cancel{g} = \cancel{g}(Tt - \frac{1}{2}t^2)$

$\Rightarrow \frac{1}{2}t^2 - Tt - \frac{3}{2}T^2 \quad (\times 2) \quad t^2 - 2Tt - 3T^2 = 0$

$(t - 3T)(t + T) = 0$

$\therefore t = 3T$

5.

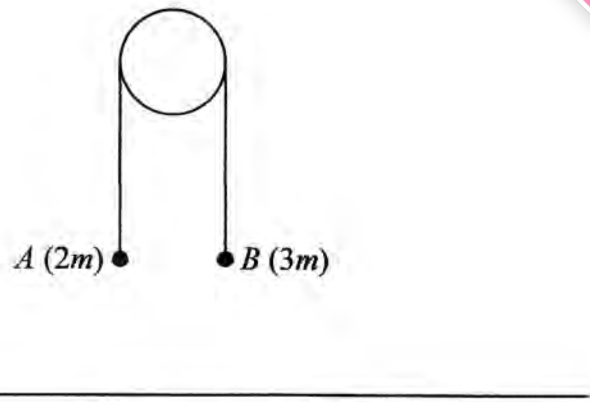


Figure 2

Two particles A and B have masses $2m$ and $3m$ respectively. The particles are connected by a light inextensible string which passes over a smooth light fixed pulley. The system is held at rest with the string taut. The hanging parts of the string are vertical and A and B are above a horizontal plane, as shown in Figure 2. The system is released from rest.

(a) Show that the tension in the string immediately after the particles are released is $\frac{12}{5}mg$.

(6)

After descending 1.5 m, B strikes the plane and is immediately brought to rest. In the subsequent motion, A does not reach the pulley.

(b) Find the distance travelled by A between the instant when B strikes the plane and the instant when the string next becomes taut.

(6)

Given that $m = 0.5$ kg,

(c) find the magnitude of the impulse on B due to the impact with the plane.

(2)

a) T T a a $2mg$ $3mg$

$3mg - T = 3ma$

$T - 2mg = 2ma$

$\cancel{m}g = 5\cancel{m}a$

$a = \frac{1}{5}g$

$T = 2ma + 2mg$

$T = \frac{2}{5}mg + 2mg = \frac{12}{5}mg$

b) (A) $s = 1.5$, $u = 0$, v, $a = 1.96$, t

$v^2 = u^2 + 2as$

$v^2 = 2(1.96)(1.5) = \frac{3}{5}g$

$v = \sqrt{\frac{3g}{5}}$

B hits ground, A now moves under gravity

$s = ?$
$u = \sqrt{\tfrac{3}{5}g}$
$v = 0$
$a = -g$
$t = x$

$\cancel{s = ut + \tfrac{1}{2}at^2}$

$\cancel{0 = \sqrt{\tfrac{3}{5}g}\, t - \tfrac{1}{2}gt^2}$

$\cancel{0 = t\left(\sqrt{\tfrac{3}{5}g} - \tfrac{1}{2}t\right)}$

$\cancel{\therefore \tfrac{1}{2}t = \sqrt{\tfrac{3}{5}g}}$

$\cancel{t = 2\sqrt{\tfrac{3}{5}g} \Rightarrow t = 4.85\text{sec}}$

2

$v^2 = u^2 + 2as$

$0 = \frac{3g}{5} - 2gs$

$s = 0.3\text{ m}$

$\therefore$ Total distance is $0.3 \times 2 = 0.6\text{m}$

c) Initial momentum $= 3\left(\tfrac{1}{2}\right)\sqrt{\tfrac{3}{5}g}$

final Momentum $= 0$

$\therefore$ Impulse $= 3.64\text{Ns}$

2

6.

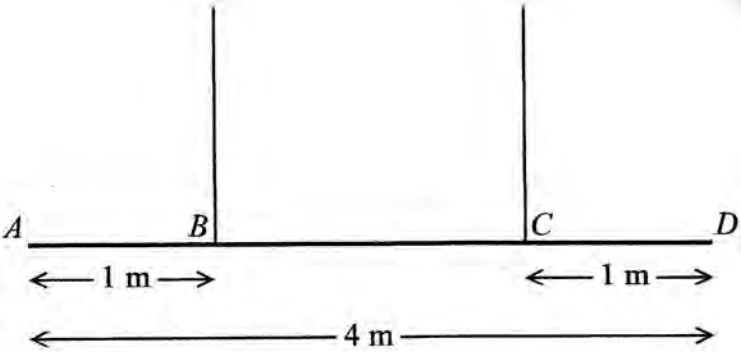


Figure 3

A non-uniform beam AD has weight W newtons and length 4 m. It is held in equilibrium in a horizontal position by two vertical ropes attached to the beam. The ropes are attached to two points B and C on the beam, where $AB = 1$ m and $CD = 1$ m, as shown in Figure 3. The tension in the rope attached to C is double the tension in the rope attached to B. The beam is modelled as a rod and the ropes are modelled as light inextensible strings.

(a) Find the distance of the centre of mass of the beam from A.

(6)

A small load of weight kW newtons is attached to the beam at D. The beam remains in equilibrium in a horizontal position. The load is modelled as a particle.

Find

(b) an expression for the tension in the rope attached to B, giving your answer in terms of k and W,

(3)

(c) the set of possible values of k for which both ropes remain taut.

(2)

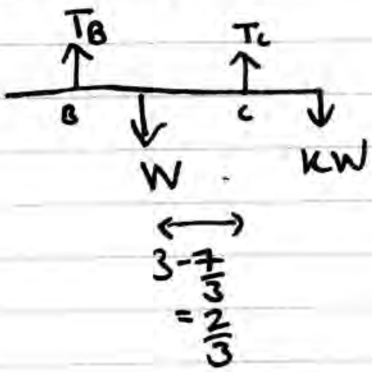
a)

$\uparrow = \downarrow \Rightarrow 3T = W$

$A \circlearrowright \; T \times 1 + 2T \times 3 = Wx$

$7\cancel{T} = 3\cancel{T}x \quad \therefore x = \frac{7}{3}\text{m}$

b)



$C \circlearrowright \; kW \times 1 + T_B \times 2 = W \times \frac{2}{3}$

$2T_B = \frac{2}{3}W - kW$

$T_B = \frac{1}{3}W - \frac{1}{2}kW$

c) Cannot tilt about B

tilting about C when $T_B = 0$ when u max valu

$C \circlearrowleft \quad u_{max} \cancel{W} \times 1 = \frac{2}{3} \cancel{W}$

$\therefore \; u_{max} = \frac{2}{3}$

$0 < k \leq \frac{2}{3}$

7.

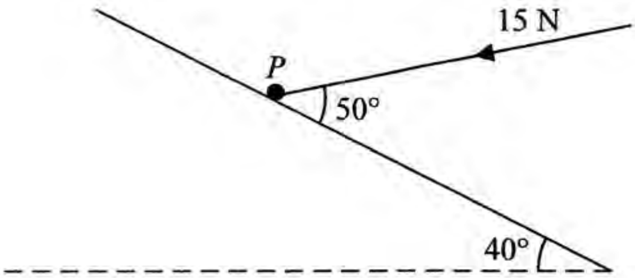


Figure 4

A particle P of mass 2.7 kg lies on a rough plane inclined at 40° to the horizontal. The particle is held in equilibrium by a force of magnitude 15 N acting at an angle of 50° to the plane, as shown in Figure 4. The force acts in a vertical plane containing a line of greatest slope of the plane. The particle is in equilibrium and is on the point of sliding down the plane.

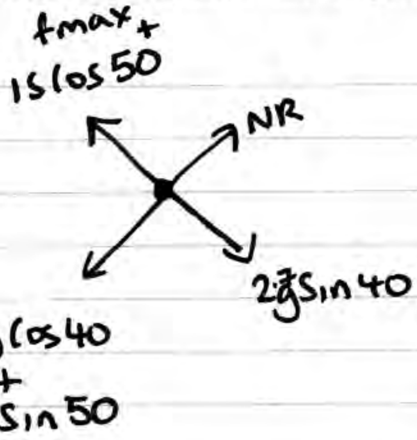
Find

(a) the magnitude of the normal reaction of the plane on P,
(4)

(b) the coefficient of friction between P and the plane.
(5)

The force of magnitude 15 N is removed.

(c) Determine whether P moves, justifying your answer.
(4)

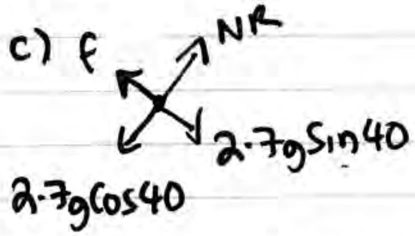


a) $NR = 2.7g\cos 40 + 15\sin 50$
$= 31.8$

b) $f_{max} = \mu NR = 31.8\mu$

$R(\nwarrow) = 0 \quad 31.8\mu + 15\cos 50 = 2.7g\sin 40$
$31.8\mu = 7.366$
$\mu = 0.23$

c)



If $2.7g\sin 40 > f_{max}$
P will move down the plane

$2.7g\sin 40 = 17.008$

$f_{max} = \mu NR$
$= 0.23(2.7g\cos 40)$
$= 4.7$

$\therefore$ It will move!